Optimal Sustainable Pension Investing

Continuous-Time Defined-Contribution Portfolio Choice with ESG Preferences

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Why this matters now

Defined contribution pensions are now central

- DC plans increased from **48.4%** of P7 pension assets in 2017 to around **63%** in 2026.[1]
- DC assets grew by **9.4%** per year, compared with **3.2%** for DB assets.[1]

Sustainability is now part of institutional investing

- PRI signatories increased from **3,038** in 2020 to **5,261** in 2025.[2]
- PRI signatory AUM rose from **USD 103.4 trillion** to **USD 139.6 trillion**.[2]

[1] Thinking Ahead Institute (2017, 2026).
[2] Principles for Responsible Investment (2020, 2025).

Literature and research gap

- **DC pension optimisation:** a large literature studies dynamic allocation under contributions, guarantees, wage risk and interest-rate risk.[1]
- **Sustainable portfolio choice:** ESG preferences, ESG scores and ESG-efficient frontiers are well studied, but mainly in static settings.[2]
- **Dynamic ESG and DC pensions:** fewer papers study ESG in dynamic portfolio choice, and DC applications remain mostly asset-label based.[3]

Research gap

To the best of our knowledge, no tractable continuous-time DC model embeds a **portfolio-level ESG target** directly into the allocation problem.

This paper introduces a portfolio-level ESG penalty into the DC portfolio-choice problem.

[1] Boulier et al. (2001); Vigna and Haberman (2001); Battocchio and Menoncin (2004); Cairns et al. (2006); Menoncin and Vigna (2017).
[2] Dorfleitner and Nguyen (2017); Pedersen et al. (2021); Abate et al. (2024).
[3] Peng et al. (2024); Buffoli, Menoncin and Rolfi, *Revise & Resubmit*, *Quantitative Finance*.

Model setup: market and contributions

- One risk-free asset:

$$dB(t) = rB(t)\,dt.$$

- N risky assets:

$$dS(t) = I_S(t)\mu\,dt + I_S(t)\Sigma\,dW(t).$$

- Labour income grows deterministically:

$$dL(t) = gL(t)\,dt.$$

- $\psi_S \in \mathbb{R}^N$ denotes the vector of risky-asset ESG scores, while $\psi_B \in \mathbb{R}$ denotes the ESG score of the risk-free asset.

Model: the DC accumulation channel

Total pension wealth

$$X(t) = \underbrace{F(t)}_{\text{financial wealth}} + \underbrace{C(t)}_{\text{contribution capital}}.$$

Contribution capital

$$C(t) = \int_t^T \kappa L(s) e^{-r(s-t)}\, ds = \kappa L(t) \frac{1 - e^{-(r-g)(T-t)}}{r-g}.$$

Portfolio weights

$$\omega(t)^\top = \frac{\theta(t)^\top I_S(t)}{X(t)}, \qquad \omega_B(t) = 1 - \omega(t)^\top \mathbf{1}.$$

Contributions capitalise into $C(t)$; total wealth $X(t) = F(t) + C(t)$ then follows clean portfolio dynamics.

Total wealth

$$dX(t) = X(t)\left[r + \omega(t)^\top (\mu - r\mathbf{1})\right] dt + X(t)\omega(t)^\top \Sigma\, dW(t).$$

Objective: financial wealth meets portfolio ESG

Portfolio ESG score

$$\psi_X(t) = \omega(t)^\top \psi_S + \omega_B(t)\psi_B.$$

Optimisation problem

$$\sup_{\omega} \mathbb{E}_t \left[\underbrace{\chi e^{-\rho(T-t)} \ln X(T)}_{\text{terminal pension wealth}} - \underbrace{(1-\chi) \int_t^T e^{-\rho(s-t)} \left(\psi_X(s) - \Psi\right)^2 ds}_{\text{cumulative ESG deviation}} \right].$$

- χ controls the trade-off between terminal wealth and ESG alignment.
- The ESG target Ψ is imposed on the **portfolio score**, as a soft penalty.

Main result: a closed-form allocation rule

Define

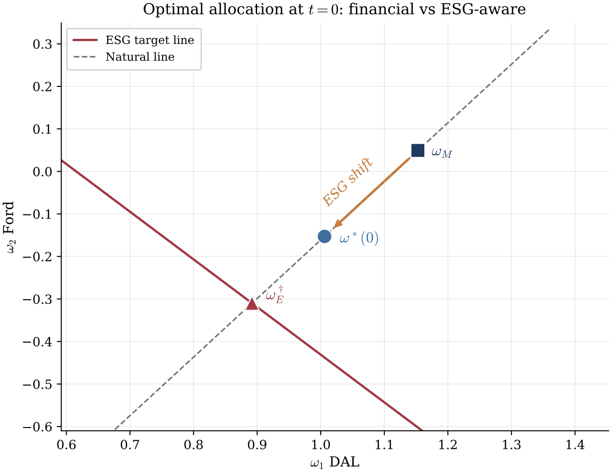
$$\xi = \psi_S - \psi_B \mathbf{1}, \qquad \zeta = \Psi - \psi_B, \qquad q = \xi^\top A^{-1} \xi \geq 0.$$

$$\boxed{\omega^{*\top} = \underbrace{\omega_M^\top}_{\text{Merton benchmark}} + \frac{2(1-\chi)(\zeta - \omega_M^\top \xi)}{1 + 2(1-\chi) q} \underbrace{\xi^\top A^{-1}}_{\text{ESG direction}}}$$

- ω_M: the financial benchmark, obtained when the ESG term is absent.
- $\zeta - \omega_M^\top \xi$: the ESG gap of the Merton portfolio relative to the target.
- $\xi^\top A^{-1}$: the risk-adjusted direction along which the ESG mandate tilts the allocation.
- $A \propto \Sigma\Sigma^\top$: the asset risk (covariance) matrix; $q = \xi^\top A^{-1} \xi \geq 0$ measures ESG dispersion in that risk metric.

Geometry of the ESG adjustment

Two-risky-asset illustration: ω_1 = Delta Air Lines, ω_2 = Ford.

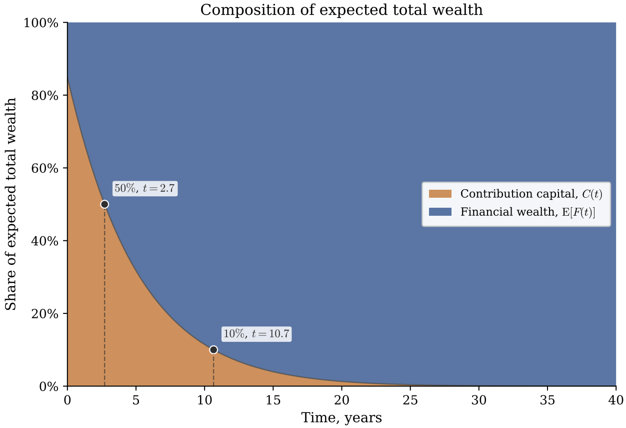


- ω_M: the financial benchmark, obtained when the ESG term is absent.
- $\zeta - \omega_M^\top \xi$: the ESG gap of the Merton portfolio relative to the target.
- $\xi^\top A^{-1}$: the risk-adjusted direction along which the ESG mandate tilts the allocation.
- A: the risk-geometry matrix induced by the covariance structure of risky returns; $q = \xi^\top A^{-1} \xi \geq 0$ measures ESG dispersion in that metric.

Numerical illustration: baseline calibration

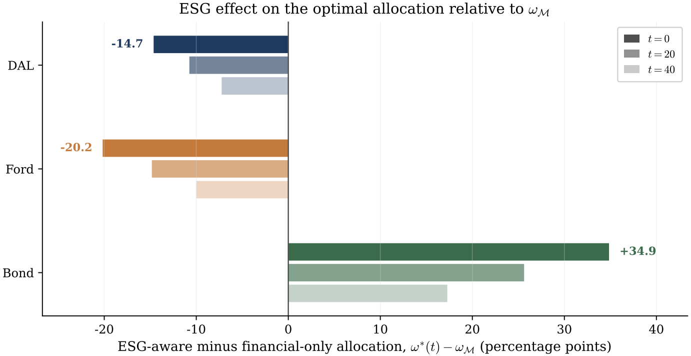
Parameter	Value	Source / interpretation
Financial market		
Risky assets	Delta Air Lines, Ford	
Return sample	2010–2025	
Risk-free rate	$r = 1.21\%$	3-month U.S. Treasury Bill proxy.
DC pension fund		
Accumulation horizon	$T = 40$	
Contribution rate	$\kappa = 12\%$	Vanguard *How America Saves 2025*.
Wage growth	$g = 3.61\%$	U.S. Average Wage Index trend.
Sustainability inputs		
Risky-asset ESG scores	$\psi_{DAL} = 0.35, \ \psi_{Ford} = 0.42$	S&P Global Sustainable1.
Risk-free ESG score	$\psi_B = 1.00$	Assumption.
ESG target	$\Psi = 0.60$	Assumption.
Preference parameter	$\chi = 0.95$	Assumption.

Result 1: contribution capital is economically large early on



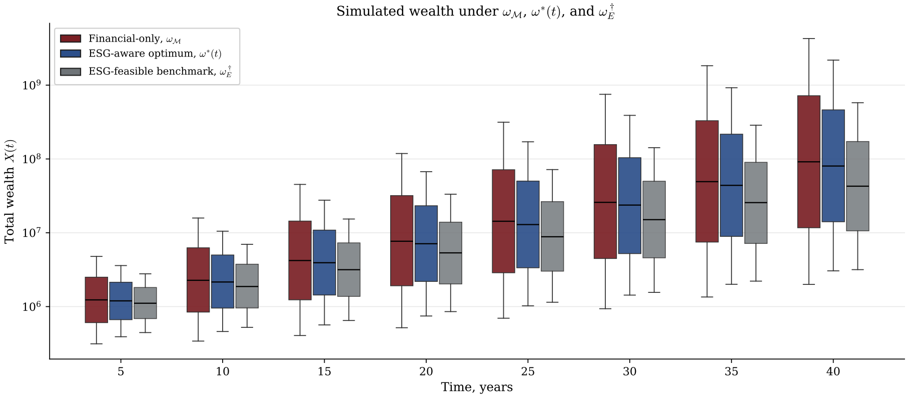
- At inception, contribution capital accounts for more than **80%** of total pension wealth.
- It falls to **50%** after about **2.7 years**.
- It falls to **10%** after about **10.7 years**.
- Financial wealth then becomes dominant through compounding.

Result 2: ESG preferences reallocate the portfolio



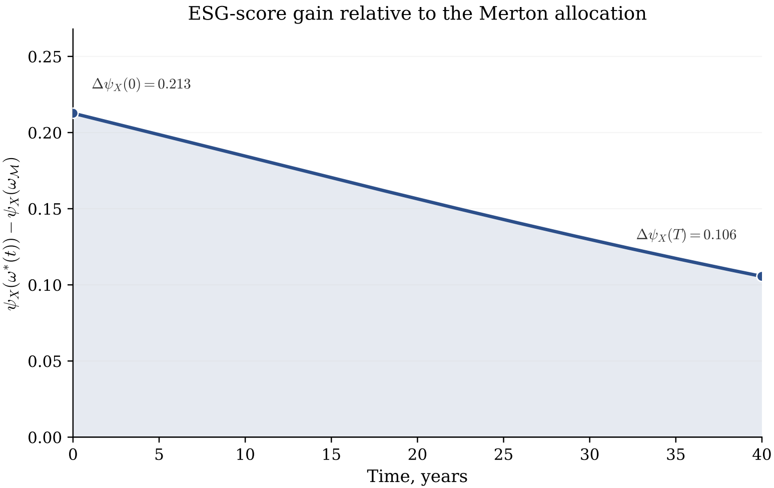
- At $t = 0$, ESG preferences lower exposure to both risky assets.
- DAL: **-14.7 pp**; Ford: **-20.2 pp**.
- The risk-free allocation rises by **+34.9 pp**.
- The shift becomes smaller as the fund approaches maturity.
- In this calibration, the tilt points toward the risk-free asset because ψ_B is highest.

Result 3a: ESG alignment has a measurable wealth cost



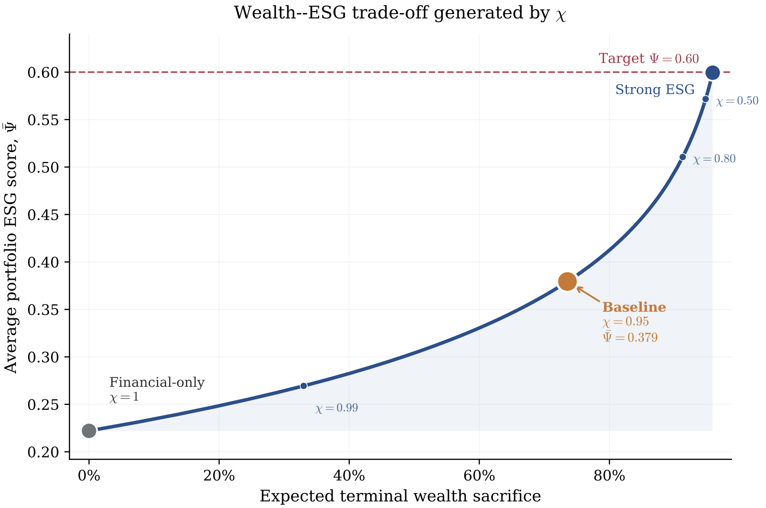
- ω_M: highest wealth.
- $\omega^*(t)$: lower wealth.
- $\omega_E^\dagger$: lowest wealth.

Result 3b: ESG alignment improves the portfolio score



- The ESG-aware policy delivers a positive portfolio-score gain relative to Merton.
- Near maturity, terminal-wealth utility dominates the ESG adjustment.

Result 4: the wealth–ESG trade-off becomes operational



- $\chi = 1$: financial-only benchmark.
- Lower χ: higher portfolio ESG score.
- ESG improvement requires an expected terminal-wealth sacrifice.
- Governance interpretation: choose a wealth-loss budget, then recover the strongest compatible ESG preference.

What the paper contributes

1. **Portfolio-level ESG preferences.** Sustainability enters the objective through a quadratic penalty on deviations from a portfolio ESG target.
2. **Closed-form and geometric solution.** The optimal allocation decomposes into a Merton benchmark plus one ESG-driven adjustment, with a clear geometric interpretation.
3. **Explicit wealth–sustainability trade-off.** The model quantifies how much terminal wealth is sacrificed to improve the portfolio ESG score.
4. **Governance interpretation of ESG preference.** The preference parameter χ can be linked to an acceptable expected terminal-wealth loss.

Thank you

Questions?

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Personal website

Backup slides

Available for Q&A

Backup: financial wealth and contribution capital

Financial wealth

$$dF(t) = \Big[rF(t) + \theta(t)^\top I_S(t)(\mu - r\mathbf{1}) + \kappa L(t)\Big]\,dt + \theta(t)^\top I_S(t)\Sigma\, dW(t).$$

Contribution capital

$$C(t) = \int_t^T \kappa L(s) e^{-r(s-t)}\,ds = \kappa L(t)\frac{1 - e^{-(r-g)(T-t)}}{r-g}.$$

$$dC(t) = \big(rC(t) - \kappa L(t)\big)\,dt.$$

Total pension wealth

$$X(t) = F(t) + C(t).$$

$$dX(t) = X(t)\big[r + \omega(t)^\top(\mu - r\mathbf{1})\big]\,dt + X(t)\omega(t)^\top \Sigma\, dW(t).$$

Backup: HJB structure

Dynamic programming equation

$$0 = J_t - \rho J + \max_{\omega} \left\{ \mathcal{L}^{\omega} J - (1-\chi)(\psi_X - \Psi)^2 \right\}.$$

Log-utility ansatz

$$J(t,x) = H(t)\ln x + B(t), \qquad H(t) = \chi e^{-\rho(T-t)}.$$

Backup: convex-combination interpretation

In the non-degenerate case $q > 0$,

$$\omega^{*\top} = \frac{1}{1 + 2q(1-\chi)} \omega_M^\top + \frac{2q(1-\chi)}{1 + 2q(1-\chi)} \omega_E^{\dagger\top}.$$

Parameter	Effect on ω^*	Intuition
$\chi = 1$	$\omega^* = \omega_M$	Financial-only problem
$\chi \downarrow$	closer to $\omega_E^\dagger$	stronger ESG preference
$q \uparrow$	stronger ESG displacement	more ESG dispersion in risk metric

Backup: the role of the risk-free ESG score

- ESG scores enter the allocation through $\xi = \psi^S - \psi_B \mathbf{1}$ and $\zeta = \boldsymbol{\Psi} - \psi_B$.
- The baseline sets $\psi_B = 1$ as a modelling normalisation.
- This choice explains why, in the numerical example, ESG alignment increases the risk-free position.
- The mechanism is more general. If a risky asset had a sufficiently high ESG score, the same rule could tilt toward that risky asset.
- ESG-rating disagreement can be represented as uncertainty in ξ, suggesting a natural robustness extension.

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