NatPar: Natural Parametric Modeling

A technical reporting language for parametric natural catastrophe insurance to make it comparable with NatCat portfolio and auditable for capital

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Two views of parametric insurance

The gap this paper addresses

Academic — design-first

Choose payout $I(\cdot)$ on an index X:

$$\max_{I} \mathcal{U}\Big(W_0 - \pi + I(X) - L\Big) \ \text{ s.t. } \ \pi = (1+\theta)\,\mathbb{E}[I(X)].$$

Elegant; basis risk $=$ mismatch $L - I(X)$. But an arbitrary $I(X)$ is not how products are *built or governed*.

Industry — pipeline-first

Products live inside the NatCat supply chain:

$$\underbrace{\text{Hazard} \to \text{Vuln.} \to \text{Fin.}}_{\text{exposure feeds in}}$$

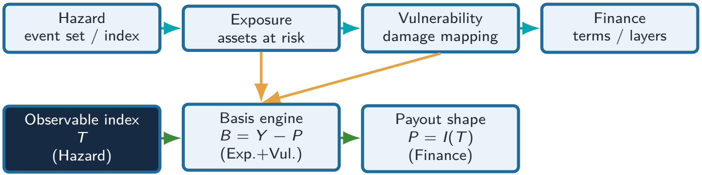
$$\Rightarrow \ \{\text{AAL},\ \text{EP}\,/\,\text{AEP}\,/\,\text{OEP},\ \text{tails}\}.$$

A contract must be *verifiable, auditable, comparable* in this same language.

The gap becomes acute when products must be justified to boards, supervisors, and capacity providers.

This paper: a *pipeline-first* definition — triggers, payouts and validation designed to be NatCat-compatible, with basis risk as the measurable interface created when (exposure $\times$ vulnerability) is replaced by an index proxy.

NatCat stack to NatPar overlay



Question

How can a parametric NatCat product be reported so that its liability tail and basis risk are comparable with an indemnity NatCat benchmark?

1. Pipeline standard

Same hazard-exposure-vulnerability-finance architecture as NatCat.

2. Distributional basis

Basis risk is a random variable and an exceedance curve, not one average.

3. Capital comparison

AAL-neutrality is not capital-neutrality; payout shape can dominate VaR/TVaR.

Technical object of the talk

$$Y \text{ (indemnity loss)}, \qquad P = I(T) \text{ (parametric payout)}, \qquad B = Y - P \text{ (basis)}$$

NatCat benchmark: stochastic portfolio objects

Let Y_r be the contractual NatCat loss for risk/region $r \in \mathcal{R}$. Define

$$M = \max_{r \in \mathcal{R}} Y_r, \qquad S = \sum_{r \in \mathcal{R}} Y_r.$$

Exceedance-probability curves

$$\begin{aligned} \mathrm{OEP}(x) &= \mathbb{P}(M > x), \\ \mathrm{AEP}(x) &= \mathbb{P}(S > x), \qquad x \geq 0. \end{aligned}$$

One-year capital metrics

$$\begin{aligned} \mathrm{VaR}_\alpha(X) &= \inf\{x : F_X(x) \geq \alpha\}, \\ \mathrm{TVaR}_\alpha(X) &= \mathbb{E}[X \mid X > \mathrm{VaR}_\alpha(X)]. \end{aligned}$$

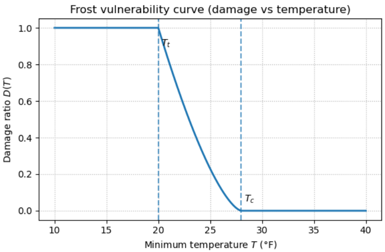
Return level

$$x_T(X) = \inf\{x : \mathbb{P}(X > x) \leq 1/T\}.$$

Compatibility principle

NatPar should be reported on the same AEP / OEP, return-period, and capital scale as the indemnity benchmark.

Payout classes and expected tail geometry



Damage ratio $D(T)$ as a function of minimum temperature.

Define

$$\begin{aligned} T &: \text{seasonal minimum temperature}, \\ D &= D(T) \in [0,1], \\ A &: \text{exposure/severity multiplier}, \\ L &= AD. \end{aligned}$$

Class	**Prototype**	**Technical implication**
Continuous bounded	$P = q(D-d)^+,\ D \in [0,1]$	Smooth response above trigger; liability cap $q(1-d)$; diffuse basis mismatch.
Tiered / piecewise	$P = \sum_k q_k \mathbf{1}\{d_k < D \le d_{k+1}\}$	Transparent breakpoints; can target return levels or VaR constraints.
Binary trigger	$P = q\mathbf{1}\{D > d\}$	Discrete distribution; EP curve has a plateau; VaR can jump at regulatory quantiles.
Layer-like parametric	capped linear or multi-step schedules	Closer to indemnity layers; can match mean and selected tail constraints.

Design diagnostic

The payout map $I(\cdot)$ determines not only $\mathbb{E}[P]$, but the full shape of $\mathbb{P}(P > x)$, $\mathbb{P}(Y - P > x)$, and the capital comparison.

AAL-neutral calibration: useful baseline, weak tail control

Let Y be the indemnity benchmark on a common hazard/exposure base and let $P = I(T;\theta)$. A baseline comparison is

$$\boxed{\mathbb{E}[P] = \mathbb{E}[Y]} \qquad \text{AAL-neutrality.}$$

Why use it?

- removes mean-loss differences across shapes;
- isolates tail changes attributable to $I(\cdot)$ and basis structure;
- approximates pure-risk-premium comparison before loadings.

What it does not imply

$$\mathbb{E}[P] = \mathbb{E}[Y] \not\Rightarrow \mathrm{VaR}_{0.995}(P) = \mathrm{VaR}_{0.995}(Y),$$
$$\mathbb{E}[P] = \mathbb{E}[Y] \not\Rightarrow \mathrm{BEP}^{+}(x) \text{ is small in the tail.}$$

The experiment uses AAL-neutrality as a microscope: suppress the mean effect so distribution-shape and non-spanning effects are visible.

Basis risk as the auditable interface

Define basis for a policy/region as $B := Y - P$, with positive sign meaning policyholder shortfall.

Shortfall

$B > 0$
indemnity loss exceeds parametric payout

Matched

$B = 0$
payout matches benchmark

Overpayment

$B < 0$
parametric payout exceeds benchmark

Point diagnostics

$$\mathbb{E}[B^+], \quad \mathbb{E}[B^-], \quad \mathbb{P}(B > 0), \quad \mathbb{P}(B < 0), \quad \mathrm{Corr}(Y, P).$$

Under AAL-neutrality

$$\mathbb{E}[B] = 0 \quad \Rightarrow \quad \mathbb{E}[B^+] = \mathbb{E}[B^-].$$

Mean balance can hide asymmetric tails.

Basis risk should be reported as two tail functions rather than a scalar correlation.

Shortfall basis EP

$$\mathrm{BEP}^+(x) = \mathbb{P}(B > x), \qquad x \geq 0.$$

Policyholder/conduct side: how often the indemnity loss exceeds the payout by more than x.

Overpayment basis EP

$$\mathrm{BEP}^-(x) = \mathbb{P}(-B > x), \qquad x \geq 0.$$

Issuer pricing/capital side: how often the payout exceeds the benchmark by more than x.

Basis return levels

$$b_T^+ = \inf\{x : \mathrm{BEP}^+(x) \leq 1/T\}, \qquad b_T^- = \inf\{x : \mathrm{BEP}^-(x) \leq 1/T\}.$$

Portfolio basis: aggregate and occurrence views

For region-level basis $B_r = Y_r - P_r$, define

$$S_B = \sum_{r \in \mathcal{R}} B_r, \qquad M_B = \max_{r \in \mathcal{R}} B_r.$$

Basis aggregate EP

$$\mathrm{BAEP}^{\pm}(x) = \mathbb{P}(\pm S_B > x).$$

Measures total portfolio mismatch and diversification effects.

Basis occurrence EP

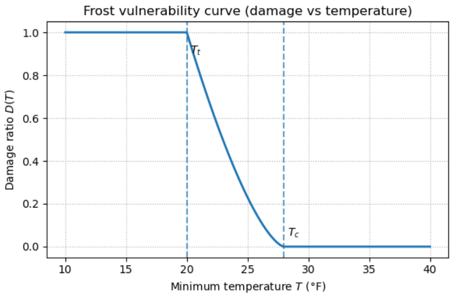
$$\mathrm{BOEP}^{\pm}(x) = \mathbb{P}(\pm M_B > x).$$

Measures worst-region mismatch; useful for suitability and concentration diagnostics.

Governance interpretation

BEP^{+}, BAEP^{+}, and BOEP^{+} are policyholder shortfall tails; the negative-side curves represent overpayment/capital tails.

Frost laboratory: model ingredients



Damage ratio $D(T)$ as a function of minimum temperature.

For region $r \in \{\mathrm{FL}, \mathrm{CA}\}$, define

$$\begin{aligned} T_r &: \text{seasonal minimum temperature}, \\ D_r &= D_r(T_r) \in [0,1], \\ A_r &: \text{exposure/severity multiplier}, \\ L_r &= A_r D_r. \end{aligned}$$

Layer benchmark

$$l_{r,\beta} = \mathrm{VaR}_\beta(L_r), \qquad Y_{r,\beta} = (L_r - l_{r,\beta})^+.$$

Frost laboratory: triggers and controlled comparison

Parametric trigger threshold is selected on the damage scale:

$$d_{r,\beta} = \mathrm{VaR}_\beta(D_r), \qquad \beta \in \{0.90, 0.95, 0.99\}.$$

NatCat benchmark

$$Y_{r,\beta} = (A_r D_r - l_{r,\beta})^+.$$

Loss layer depends on damage D_r and exposure severity A_r.

NatPar candidate

$$P_{r,\beta} = f_{r,\beta}(D_r).$$

The payout observes only the damage/index coordinate.

Controlled comparison

Each NatPar design is calibrated so that

$$\mathbb{E}[P_{r,\beta}] = \mathbb{E}[Y_{r,\beta}]$$

within each (r, β) block.

Analytic EP reduction for the indemnity benchmark

Assume A_r and D_r are independent and write $\overline{F}_{A,r}(a) = \mathbb{P}(A_r > a)$. For $Y_{r,\beta} = (A_r D_r - l_{r,\beta})^+$:

Benchmark EP curve

$$\mathbb{P}(Y_{r,\beta} > x) = \mathbb{E}\left[\overline{F}_{A,r}\left(\frac{l_{r,\beta} + x}{D_r}\right) \mathbf{1}_{\{D_r > 0\}}\right]$$

Statistical role

The NatCat tail is driven by both D_r and the right tail of A_r. A D-only NatPar payout cannot span all states of Y.

Analytic AAL reduction for the indemnity benchmark

The same conditional argument gives a closed-form target for AAL-neutral calibration.

Benchmark AAL

$$\mathrm{AAL}_{r,\beta}^{\mathrm{Cat}} = \mathbb{E}[(A_r D_r - l_{r,\beta})^+] = \mathbb{E}\left[D_r \int_{l_{r,\beta}/D_r}^{\infty} \overline{F}_{A,r}(u)\ \mathrm{d}u\ \mathbf{1}_{\{D_r>0\}} \right]$$

Use in the experiment

$$\mathbb{E}[P_{r,\beta}] = \mathrm{AAL}_{r,\beta}^{\mathrm{Cat}}$$

so differences in EP, VaR, TVaR, and $\mathrm{BEP}^{\pm}$ are not mean-loss artifacts.

AAL-neutral design I: continuous bounded payout

Definition and calibration

$$P^c_{r,\beta} = q^c_{r,\beta}(D_r - d_{r,\beta})^+,$$
$$q^c_{r,\beta} = \frac{\mathbb{E}[Y_{r,\beta}]}{\mathbb{E}[(D_r - d_{r,\beta})^+]}.$$

Liability EP curve

$$\mathbb{P}(P^c_{r,\beta} > x) = \mathbb{P}\left(D_r > d_{r,\beta} + \frac{x}{q^c_{r,\beta}}\right).$$

Tail geometry

Continuous payout preserves severity ordering above the trigger, but is still bounded by $q^c_{r,\beta}(1 - d_{r,\beta})$.

AAL-neutral design II: binary trigger payout

Definition and calibration

$$P^b_{r,\beta} = q^b_{r,\beta}\mathbf{1}_{\{D_r > d_{r,\beta}\}},$$
$$q^b_{r,\beta} = \frac{\mathbb{E}[Y_{r,\beta}]}{\mathbb{P}(D_r > d_{r,\beta})}.$$

Liability EP curve

$$\mathbb{P}(P^b_{r,\beta} > x) = \begin{cases} \mathbb{P}(D_r > d_{r,\beta}), & 0 \le x < q^b_{r,\beta}, \\ 0, & x \ge q^b_{r,\beta}. \end{cases}$$

Tail geometry

Binary payout creates a point mass and an EP plateau; all triggered states receive the same payment.

Analytic basis EP: shortfall tail and overpayment tail

Let $P_r = f_r(D_r)$, $B_r = Y_r - P_r$, and $Y_r = (A_r D_r - l_r)^+$. For $x \ge 0$:

Shortfall tail

$$\mathrm{BEP}_r^+(x) = \mathbb{E}\left[\overline{F}_{A,r}\left(\frac{l_r + x + f_r(D_r)}{D_r}\right)\mathbf{1}_{\{D_r>0\}}\right]$$

Mechanism

Large exposure severity A_r can make Y_r large even when $f_r(D_r)$ is bounded. Thus B_r^+ can inherit the right tail of the benchmark loss.

For the same D-only payout $P_r = f_r(D_r)$, the negative-basis tail is

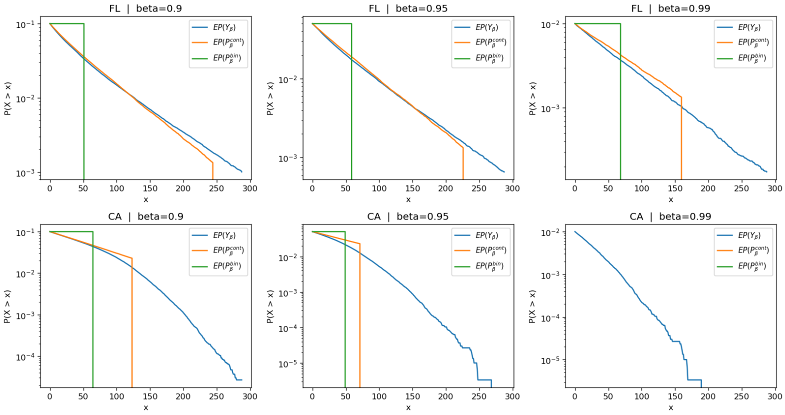
Overpayment tail

$$\mathrm{BEP}_r^-(x) = \mathbb{E}\left[F_{A,r}\left(\frac{l_r + f_r(D_r) - x}{D_r}\right)\mathbf{1}_{\{D_r>0,\ f_r(D_r)>x\}}\right]$$

Asymmetry mechanism

If $f_r(D_r)$ is bounded, then $B_r^- \le \sup_D f_r(D)$, while B_r^+ may remain unbounded or much heavier through A_r.

Individual liability tails: AAL matching does not imply EP matching



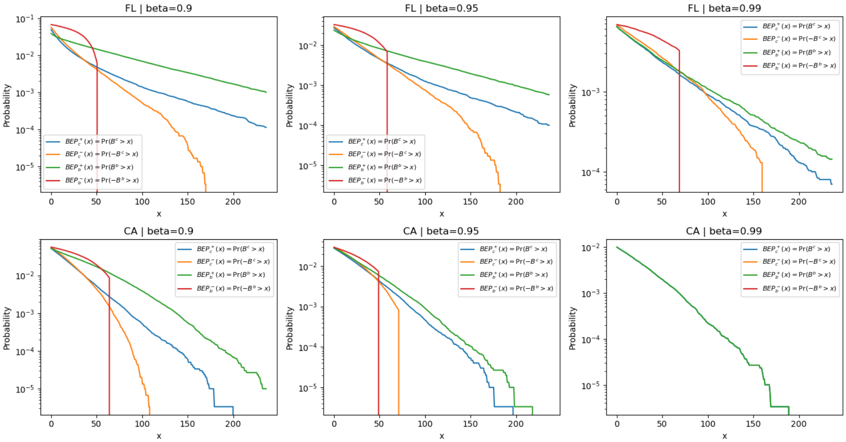
How to read panels

- row: region r;
- column: layer β;
- curves: benchmark Y, continuous P^c, binary P^b.

Finding

Binary is AAL-neutral but has a discrete capped liability tail; continuous is smoother but still cannot span exposure-driven states.

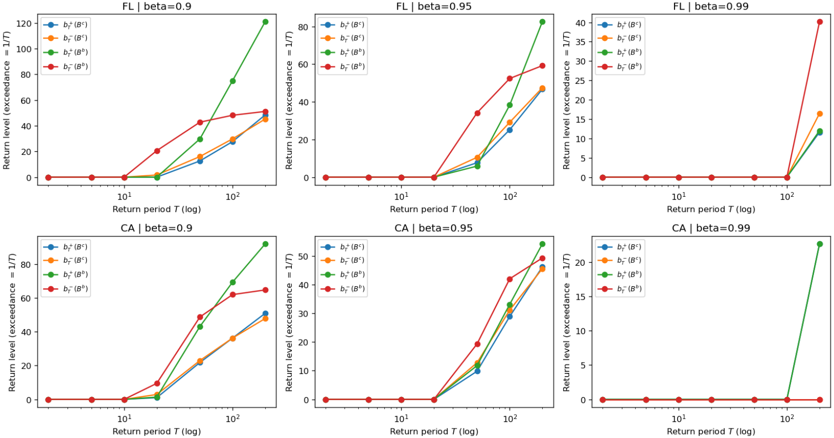
Basis exceedance curves



Reading

Shortfall and overpayment curves separate the suitability tail from the issuer-capital tail. Mean neutrality does not force either tail to be small.

Return-period basis levels



Reading

At long return periods, shortfall levels can remain large when $Y = (AD - I)^{+}$ is large because of A, while $P = f(D)$ is capped.

Individual capital at 99.5 percent

Region	β	VaR(Y)	TVaR(Y)	VaR(P^c)	TVaR(P^c)	VaR(P^b)	TVaR(P^b)
CA	0.90	156.77	184.43	123.27	123.27	64.76	64.76
CA	0.95	101.44	129.10	71.34	71.34	49.36	49.36
CA	0.99	22.65	50.31	0.00	0.00	0.00	0.00
FL	0.90	172.24	243.83	166.81	209.95	51.16	51.16
FL	0.95	141.69	213.29	143.11	189.27	59.20	59.20
FL	0.99	45.98	117.57	55.90	113.89	68.82	68.82

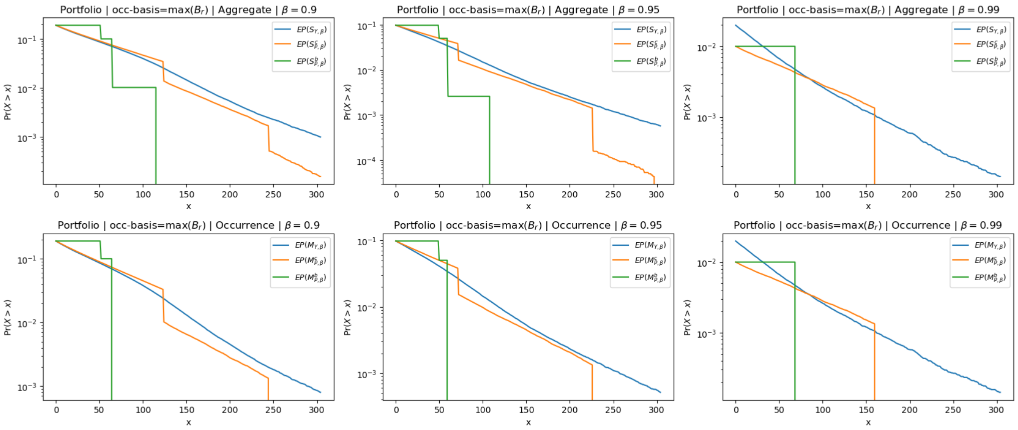
Interpretation

AAL-neutral calibration generates very different 99.5 percent liability tails.

Caution

Lower VaR(P) is not automatically a better product; it can signal non-spanned loss states and larger BEP$^+$.

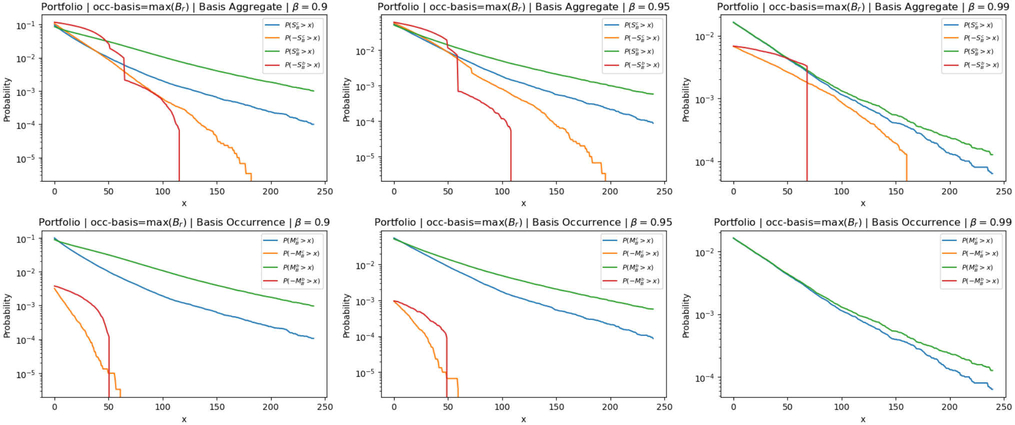
Portfolio liability reporting: AEP/OEP



Reading

AEP measures aggregate annual portfolio loss; OEP measures the largest regional occurrence in the year/event view.

Portfolio basis reporting: BAEP/BOEP



Why both views matter

BAEP measures total portfolio mismatch; BOEP measures worst-region mismatch. Diversification can coexist with a severe local basis failure.

Mechanism: unspanned exposure severity drives shortfall tails

Indemnity layer
$Y = (AD - I)^+$

→

Parametric payout
$P = f(D)$

→

Basis tail
$B = Y - P$

Mechanism I

Large A, moderate D: Y can be large while P remains small.

Mechanism II

Large D, moderate A: P responds better to damage severity.

Tail conclusion

$$B \approx Y \quad \text{when} \quad Y \gg \sup P.$$

A D-only payout is a liquidity and tractability instrument unless the index or payout incorporates exposure scaling.

Minimum viable technical reporting template

Layer	Required outputs	Governance question
Mean comparability	AAL $= \mathbb{E}[\cdot]$, calibration rule, loading basis	Is the comparison premium-neutral?
Tail comparability	EP / AEP / OEP, return levels, $\mathrm{VaR}_{0.995}$, $\mathrm{TVaR}_{0.995}$	Does the product reduce or concentrate liability tail risk?
Basis interface	$\mathbb{E}[B^+]$, $\mathbb{E}[B^-]$, $\mathrm{Corr}(Y, P)$, $\mathrm{BEP}^{\pm}$, $b_T^{\pm}$	Who is underpaid/overpaid and in which tail states?
Portfolio basis	$\mathrm{BAEP}^{\pm}$, $\mathrm{BOEP}^{\pm}$, aggregate and occurrence basis levels	Does diversification hide worst-region basis failure?
Audit trail	index source, trigger calculation, versioned payout map, climate/stress assumptions	Can the product be reproduced and challenged?

Regulatory implication

Parametric insurance can be fast-paying and capital-tractable, but suitability requires reporting what the index misses, not just what it pays.

Takeaways for a scientific and actuarial audience

1. **NatPar is a pipeline-compatible overlay:** it should inherit NatCat hazard, exposure, vulnerability, finance, and capital reporting objects.
2. **Basis risk is distributional:** report $B = Y - P$, BEP^{+}, BEP^{-}, and return-period basis levels.
3. **AAL-neutrality is not enough:** $\mathbb{E}[P] = \mathbb{E}[Y]$ does not control VaR, TVaR, or tail shortfall.
4. **Index non-spanning is the core mechanism:** D-only payouts cannot replicate exposure-amplified loss states.

Closing statement

A good NatPar product is not just fast-paying; it is transparent about what it covers, what it misses, and how those misses behave in the tail.

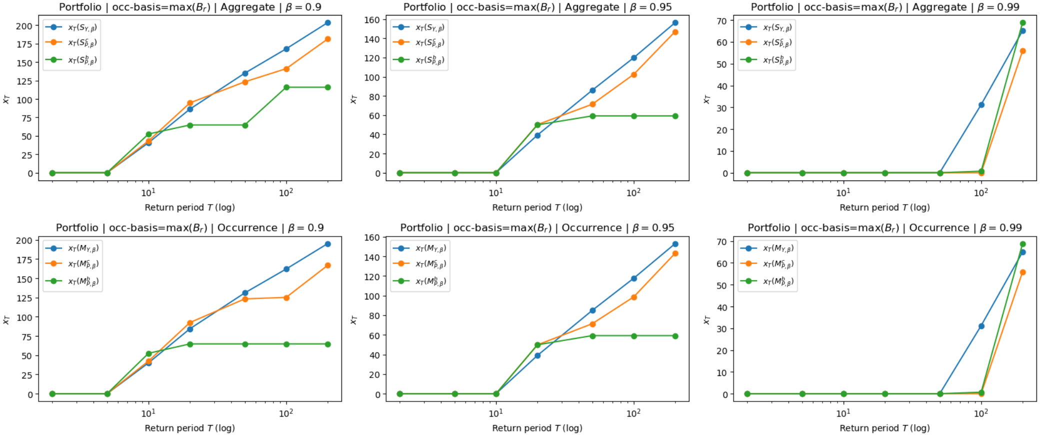
Backup: portfolio 99.5 percent capital metrics

Scope	β	VaR(Y)	TVaR(Y)	VaR(P^c)	TVaR(P^c)	VaR(P^b)	TVaR(P^b)
Agg	0.90	123.89	173.81	119.25	149.64	55.24	55.24
Agg	0.95	100.32	150.47	100.78	133.39	41.44	42.21
Agg	0.99	33.71	82.65	39.13	79.72	48.17	48.17
Occ	0.90	120.57	170.68	116.77	146.97	35.81	35.81
Occ	0.95	99.19	149.30	100.18	132.49	41.44	41.44
Occ	0.99	33.33	82.51	39.13	79.72	48.17	48.17

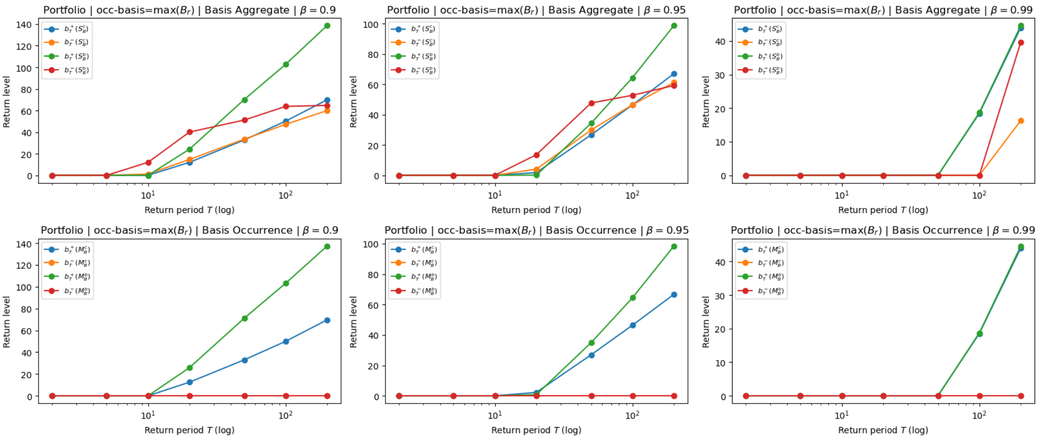
Use

Keep this slide for questions about aggregate versus occurrence capital comparisons.

Backup: portfolio liability return levels



Backup: portfolio basis return levels



Backup: extensions and open modelling questions

Multi-trigger and multi-peril NatPar

Replace scalar D by an index vector $\boldsymbol{T} = (T_1, \ldots, T_k)$ and payout $P = I(\boldsymbol{T})$. Report basis tails under dependence, cascading hazards, and compound events.

Exposure-adjusted indices

The frost mechanism suggests augmenting $f(D)$ with exposure-sensitive scaling, e.g. $P = f(D, \widehat{A})$, while preserving auditability of $\widehat{A}$.

Robust/climate-conditioned reporting

Report AEP / OEP and BEP$^{\pm}$ under scenario-conditioned hazard distributions $\mathbb{P}_s$, not only a historical-calibrated $\mathbb{P}$:

$$\sup_{s\in\mathcal{S}} \mathrm{VaR}^{(s)}_{0.995}(P), \qquad \sup_{s\in\mathcal{S}} b^{+,(s)}_{100}.$$