

A Robust Framework to Balance Anti-Discrimination and Risk-Adequacy in Insurance Pricing

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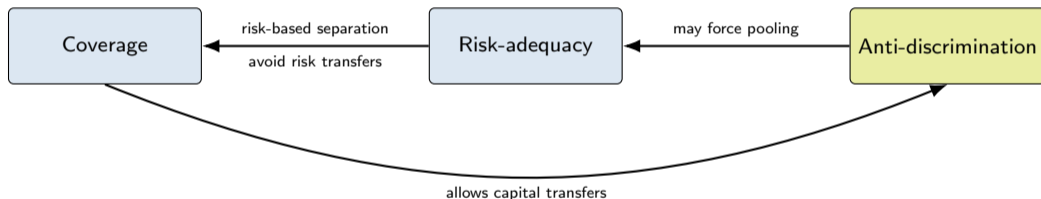
(joint work with Stefan Weber & Mario V. Wüthrich)

The basic conflict

Insurance premiums are set by pricing tariffs.

A tariff C separates risks into tariff classes. All risks within the same class pay the same price.

How to choose C ?



Idea: Quantify objectives and solve vector optimization problem for **Pareto-optimal** solutions.

$$\min_C f(C) = (f_1(C), f_2(C), \dots)$$

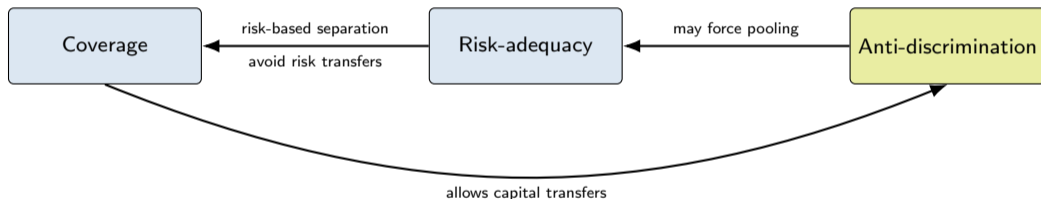
Contribution: Framework specifying the decision object, the relevant constraints, and the trade-off structure.

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What is a pricing tariff?

Policyholder level

- ▶ Each risk produces a future claim Y and is characterized by a type $\Theta = \vartheta$, that determines its claim law.
- ▶ A pricing rule Π quantifies the risk upon its claim law.

Benchmark risk premium

$$\pi(\vartheta) := \Pi(\mathbb{P}^{Y|\Theta}(\vartheta, \cdot))$$

Tariff level

\mathcal{C}	$=$	$(\mathcal{C},$	$\xi_{\Theta C},$	$\Lambda)$
		Assigns ϑ to tariff classes c	Aggregates benchmark prices inside c	Records capital transfers between classes

Charged premium

$$\pi^{\mathcal{C}}(c) := \Lambda(c) + \int \pi(\vartheta) \xi_{\Theta|C}(c, d\vartheta).$$



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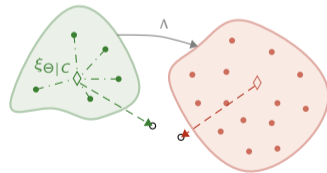
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Risk-adequate pricing - similar pay similarly, different pay differently

Avoid cross-subsidization.

- ▶ Policyholders should pay for their marginal risk contribution to the portfolio.
- ▶ Achieve suitable balance between risk-separation and pooling.

Risk-similarity: $\text{dist}(\vartheta, \vartheta') := \text{dist}_{\mathcal{M}_1}(\mathbb{P}^{\mathcal{Y}|\Theta}(\vartheta, \cdot), \mathbb{P}^{\mathcal{Y}|\Theta}(\vartheta', \cdot))$, for a probability pseudometric $\text{dist}_{\mathcal{M}_1}$.

Type-level prices r are **risk-adequate** if, for \mathbb{P} -a.e. (ϑ, ϑ') ,

$$\underline{\varphi}(\text{dist}(\vartheta, \vartheta')) \leq \text{dist}_{\mathbb{R}}(r(\vartheta), r(\vartheta')) \leq \overline{\varphi}(\text{dist}(\vartheta, \vartheta')).$$

$\underline{\varphi}, \overline{\varphi} : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$ calibrate how much premium separation is appropriate for a given risk separation.

How risk-adequate is a tariff?

Expected distance of $\pi^{\mathbf{C}}$ to the closest risk-adequate price $r \in \mathcal{R}$.

$$f_1(\mathbf{C}) := \inf_{r \in \mathcal{R}} \mathbb{E}[\text{dist}_{\mathbb{R}}(\pi^{\mathbf{C}}(C(\Theta)), r(\Theta))].$$

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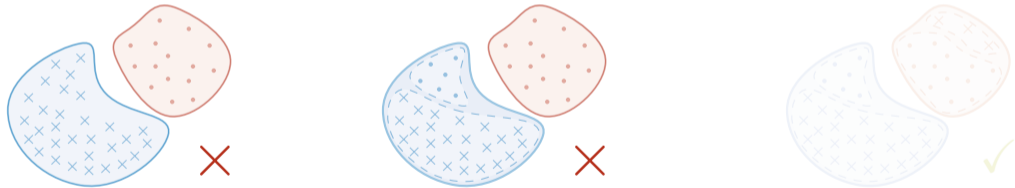
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Anti-discrimination and its effect on risk-adequacy

Anti-discrimination restricts risk-based separation.

- ▶ Some policyholder variables D are considered **protected** (e.g., gender, ethnicity, religion).
- ▶ Policyholders must not be priced differently because of D .
- ▶ No **protected group** (e.g., men, women) must be excluded from a tariff class.

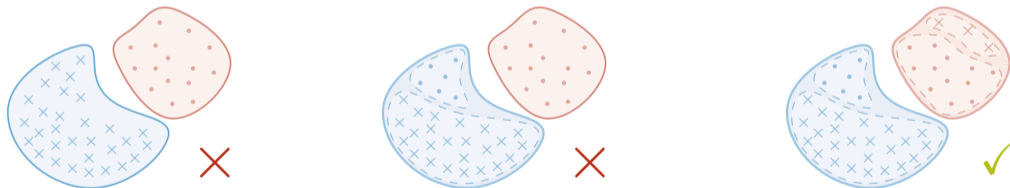


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- ▶ Unequal risks may be forced into cross-subsidization to cover portfolio risk.

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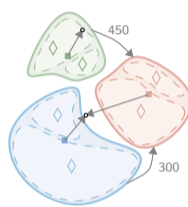
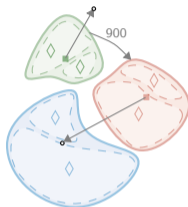
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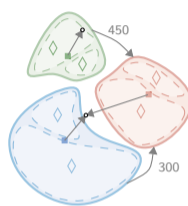
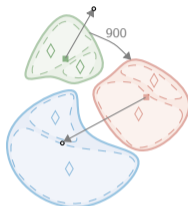
Subsidization across tariff classes could be interpreted as discrimination.

- ▶ Cross-subsidization is a capital transfer between tariff classes.
- ▶ Tariff classes C generally have different demographics $\mathbb{P}^{D|C}$.
- ▶ Transfers Λ might affect protected groups differently.

Variability of cross-subsidization among protected groups:

$$f_2(\mathbf{C}) := \text{Var}\left(\mathbb{E}[\Lambda(C) \mid D]\right)$$

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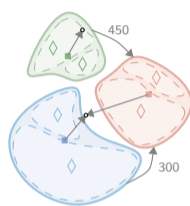
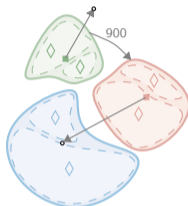
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What is new?

New perspective on anti-discriminatory pricing.

- ▶ Insurance pricing aims for portfolio coverage and two-sided risk-adequacy.
- ▶ This goal is normatively constrained by anti-discrimination law.
- ▶ Anti-discrimination is not merely a constraint on prediction, but a constraint on tariff construction.
- ▶ Anti-discrimination is decomposed into sub-objectives and quantified. Existing notions can be included.

$$\min_{\mathbf{C}} f(\mathbf{C}) = (f_1(\mathbf{C}), f_2(\mathbf{C}), \dots)$$

Robust to uncertainty. The framework can identify optimal tariffs - even if we lack portfolio information.

$$\min_{\mathbf{C}} \sup \{ f'(\mathbf{C}; \mathbb{P}); \mathbb{P} \in \mathcal{U} \}$$

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