Reverse Sensitivity Testing:

What does it take to break the model?

Silvana Pesenti

Silvana.Pesenti@cass.city.ac.uk

joint work with Pietro Millossovich and Andreas Tsanakas

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Motivation

Motivation: Real-data example

Proprietary model of a London insurance market portfolio

$$Y = g(\boldsymbol{X})$$

Facts

- 500,000 Monte Carlo simulations of inputs $\boldsymbol{X} = (X_1, \ldots, X_{72})$ and output Y
- no knowledge about distributional assumptions

Risk measures

Risk assessment of Y through the risk measures:

$$\mathsf{VaR}_\alpha(Y) = \inf\{y \in \mathbb{R} \,|\, P(Y \leq y) \geq \alpha\},$$
$$\mathsf{ES}_\alpha(Y) = \frac{1}{1-\alpha}\int_\alpha^1 \mathsf{VaR}_u(Y)\,\mathrm{d}u.$$

1. Which input factor is most important?

1. **Which input factor is most important?**
2. **Which is the most plausible alternative model that leads to an increase in the risk measure?**

Reverse sensitivity testing

Method

1. Define a **stress** on the output Y:

   - increase of VaR or/and ES

2. Derive **weights** (change of measure) such that

   - re-weighted output fulfils the required stress
   - most plausible (minimal Entropy)

3. Analyse the stressed model

   - sensitivity measure

Model and applicability

Applicable

- in a Monte Carlo setting
- for n large
- for any distribution of X_i
- for any dependence structure of $\boldsymbol{X}$
- under no restrictions on g

Monte Carlo setup

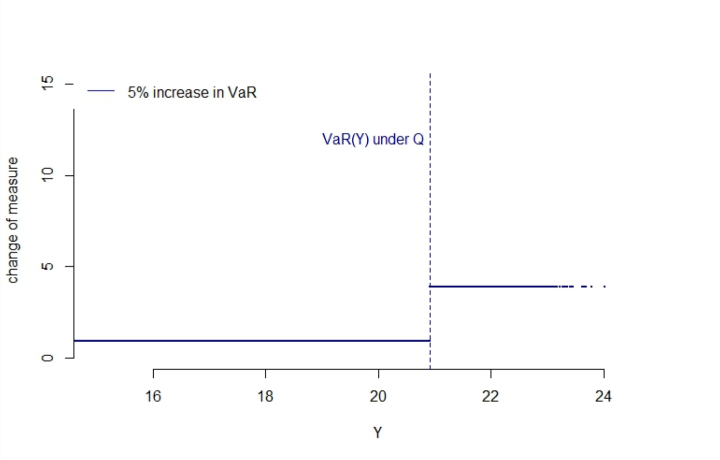
- M Monte Carlo simulations of $Y = g(\boldsymbol{X})$
- Find weights $w^{(1)}, \dots, w^{(M)}$, such that the re-weighted output has the required stress

X_1	$\dots$	X_n	Y	w
$x_1^{(1)}$	$\dots$	$x_n^{(1)}$	$y^{(1)} = g(x_1^{(1)}, \dots, x_n^{(1)})$	
$\vdots$	$\ddots$	$\vdots$	$\vdots$	?
$x_1^{(M)}$	$\dots$	$x_n^{(M)}$	$y^{(M)} = g(x_1^{(M)}, \dots, x_n^{(M)})$	

$\rightarrow$ Weights w are analytical functions of the output.

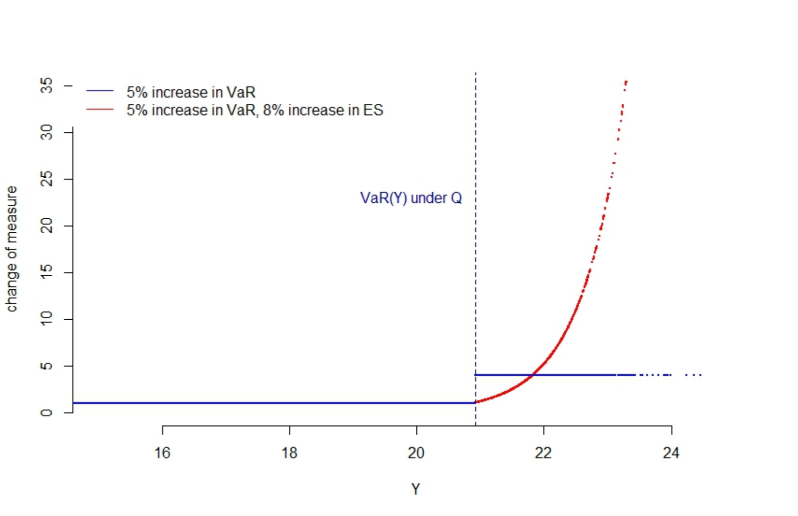
Stress on VaR

Weights for a stress on VaR



Stress on VaR and ES

Weights for a stress on VaR and ES



Numerical example

Insurance portfolio

Non-linear insurance portfolio

$$Y = L - (1 - X_4) \min \{(L - d)_+, l\}$$
$$L = X_3(X_1 + X_2),$$

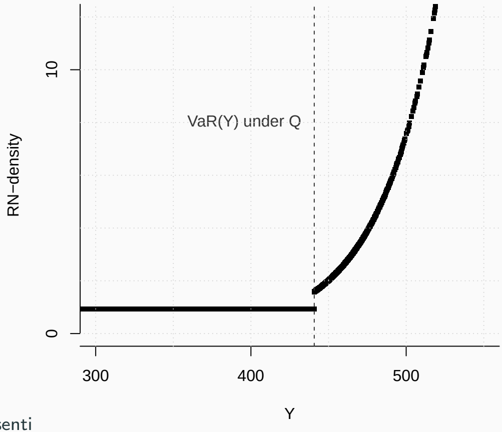
where

- X_1, X_2 different lines of business
- X_3 positive multiplicative risk factor, e.g. inflation
- X_4 percentage lost due to default of the reinsurance company
- reinsurance limit l and deductible d

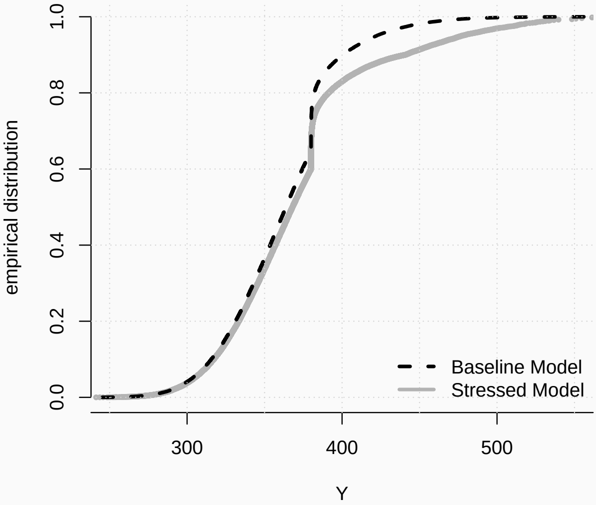
Insurance portfolio - Weights

Stress:

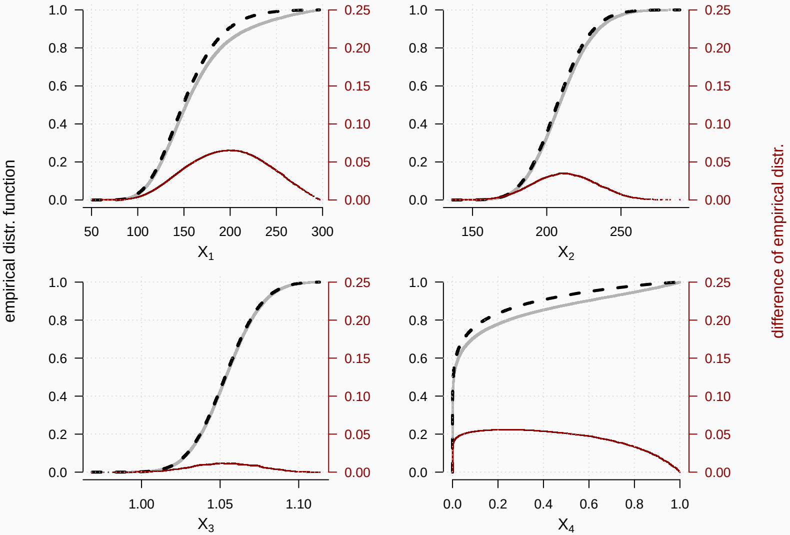
- $\text{VaR}_{0.95}(Y)$ by 10%
- $\text{ES}_{0.95}(Y)$ by 13%



Insurance portfolio - Output



Insurance portfolio - Input



Insurance portfolio

	X_1	X_2	X_3	X_4	Y
Mean	150	200	1.05	0.10	362
Mean, stressed	157	202	1.05	0.14	371
Relative increase	5%	1%	0%	44%	3%
Standard deviation	35	20	0.02	0.20	36
Standard deviation, stressed	43	21	0.02	0.26	50
Relative increase	25%	5%	1%	30%	38%

Which input factor is most important?

Sensitivity measures

Sensitivity measure

Sensitivity measure

$$\Gamma_i = \frac{E^{\text{stressed}}(X_i) - E(X_i)}{\text{normalised}}$$

- depends on Y through the weights w.

Real-data example

Proprietary model of a London insurance market portfolio

$$Y = g(\boldsymbol{X})$$

Stress:

- $\text{VaR}_{0.95}(Y)$ by 8%
- $\text{ES}_{0.95}(Y)$ by 10%

Real-data example

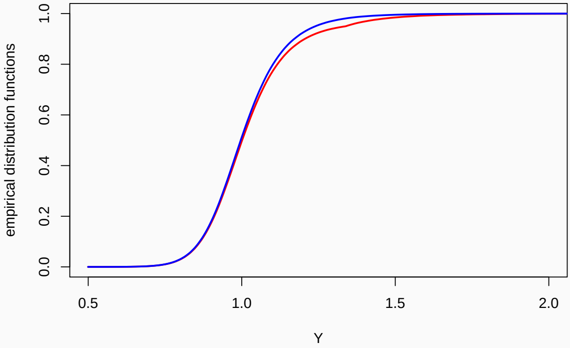
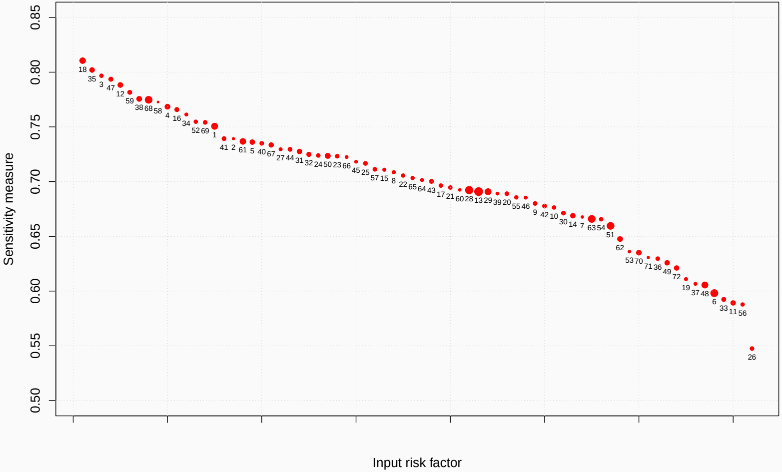


Figure 1: Empirical distribution of the output under the baseline model and the stressed model.

Real-data example



Thank you!

Appendix

Insurance portfolio - Assumptions

Assumptions:

- $X_1 \sim$ (truncated) $LogNormal$ with mean 150 and sd 35.
- $X_2 \sim Gamma$ with mean 200 and sd 20.
- $X_3 \sim$ (truncated) $LogNormal$ with mean 1.05 and sd 0.02.
- $X_4 \sim Beta$ with mean 0.1 and sd 0.2.
- X_1, X_2, X_3 are independent.
- X_4 dependent on L through a Gaussian copula with correlation 0.6.
- $d = 380$, $l = 30$.