SOLVENCY CONTAGION MODELING

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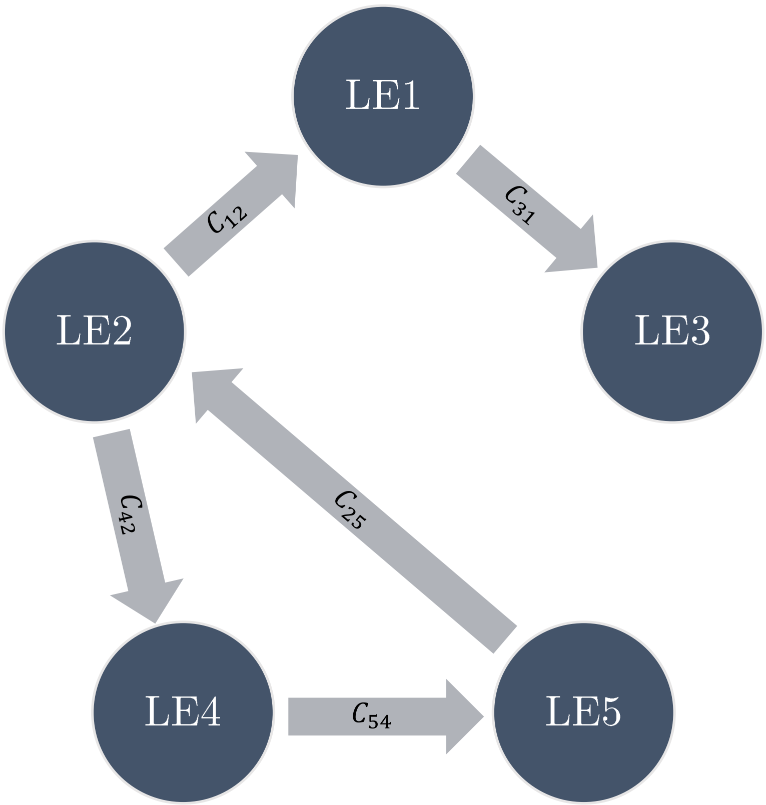
MATH. REPRESENTATION OF FINANCIAL NETWORK



LEGEND

- $i, j \in \{\text{LE1}, \ldots \text{LE5}\}$: legally separated entities
- C_{ij}: sum of capital and risk transfer instruments (CRTIs) between entities i and j, where
  i: asset holder
  j: liability holder
- C_i^e: external assets of i
- $V_{ij}(E_j): \mathbb{R} \to [0,1]$: valuation function, non-decreasing function of liability holder j's equity E_j
- Equity of entity i:

$$E_i = C_i^e + \sum_{j=1}^{n} C_{ij} V_{ij}(E_j) - \sum_{j=1}^{n} C_{ji}$$

SOLUTION: FIXED POINT OF ITERATIVE MAP

- $\boldsymbol{E}^{(k)} = [E_1^{(k)}, \ldots, E_5^{(k)}]$: vector of equities in iteration k
- *If* $\mathrm{E}_\mathrm{i}^{(0)} = \mathrm{C}_\mathrm{i}^\mathrm{e} - \sum_{\mathrm{j}=1}^{\mathrm{n}} \mathrm{C}_{\mathrm{ji}} \;\; \forall \mathrm{i}$, *a non-decreasing sequence* $\{\mathbf{E}^{(\mathrm{k})}\}$ *exists, which converges to the solution* $\mathbf{E}^{\infty} = \mathbf{E}^{-}$ (Barucca, 2016)

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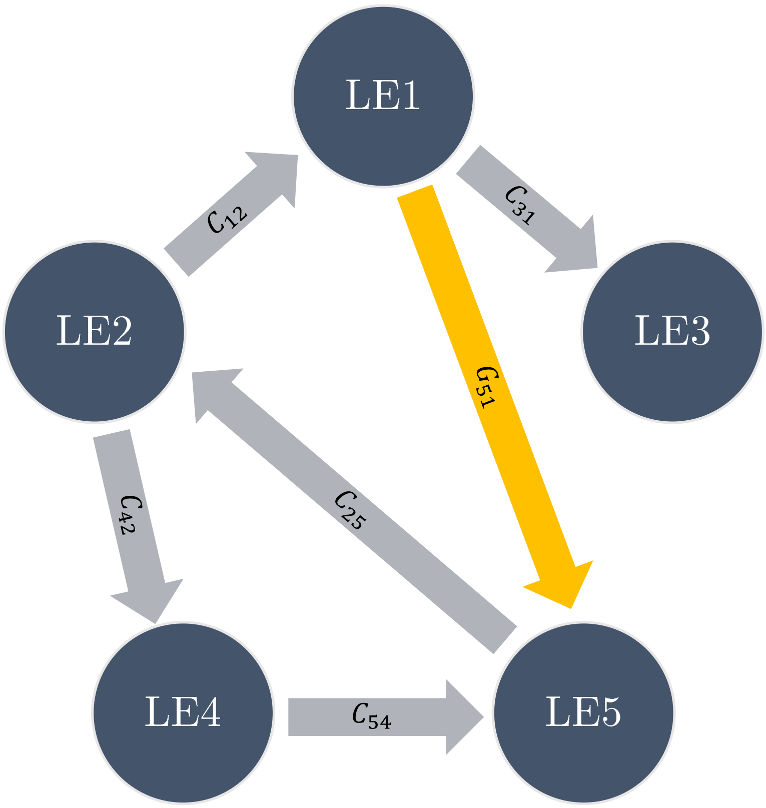

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$$E_i = C_i^e + \sum_{j=1}^{n} C_{ij} V_{ij}(E_j) - \sum_{j=1}^{n} C_{ji}$$

- $G_{ij}(E_i)$: $\mathbb{R} \to [0, G_{ij}^{max}]$: internal default guarantee liability, non-increasing function of guarantee asset holder's equity

SOLUTION: FIXED POINT OF ITERATIVE MAP

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MODIFIED PICARD ITERATION ALGORITHM

ALGORITHM WITHOUT GUARANTEES

1) $E_i^{(k+1)} = C_i^e + \sum_{j=1}^n C_{ij} v_{ij}^{(k)} - \sum_{j=1}^n C_{ji} \quad \forall i$

2) $v_{ij}^{(k+1)} = V_{ij}\left(E_j^{(k+1)}\right) \quad \forall i, j$

MODIFIED ALGORITHM WITH GUARANTEES

1) $\hat{E}_i^{(k+1)} = C_i^e + \sum_{j=1}^n C_{ij} v_{ij}^{(k)} - \sum_{j=1}^n \left(C_{ji} + G_{ji}^{(k)}\right) \quad \forall i$

2) $G_{ij}^{(k+1)} = f\left(-\hat{E}_i^{(k+1)}\right) \quad \forall i,j$

3) $E_i^{(k+1)} = \hat{E}_i^{(k+1)} + \sum_{j=1}^n G_{ij}^{(k+1)} v_{ij}^{(k)} \quad \forall i$

4) $v_{ij}^{(k+1)} = V_{ij}\left(E_j^{(k+1)}\right) \quad \forall i,j$

LEGEND

Variables, parameters, functions

- E_i: equity of entity i (incl. guarantee assets)
- $\hat{E}_i$: equity of entity i (excl. guarantee assets)
- C_i^e: external assets of i
- C_{ij}: sum of CRTIs between entities i and j, where i: asset holder; j: liability holder
- $v_{ij} = V_{ij}(E_j): \mathbb{R} \to [0,1]$: valuation
- $G_{ij}(\hat{E}_i): \mathbb{R} \to [0, G_{ij}^{max}]$: internal default guarantee liability

Sequences

- $\left\{G_{ij}^{(k)}\right\}$: non-increasing sequence
- $\left\{\hat{E}_i^{(k)}\right\}$: non-decreasing sequence
- $\left\{v_{ij}^{(k)}\right\}$: non-decreasing sequence
- $\left\{E_i^{(k+1)}\right\} = \left\{\hat{E}_i^{(k+1)} + \sum_{j=1}^n G_{ij}^{(k+1)} v_{ij}^{(k)}\right\}$: *non-decreasing sequence*

Mirai Solutions

APPLICATION

PROBLEM TO BE SOLVED FOR …

- … a large financial network of legally separated entities
- … a large-size set of independent Monte Carlo scenarios (10^5 to 10^6)
- … on a R-analytics platform

RCPP ARMADILLO

- Implementation in C++
- Problem representation is close to mathematical formulation

RCPP PARALLEL

- Monte Carlo scenarios can be run in parallel

Monte Carlo scenarios

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